EXERCISE – I SINGLE CORRECT (OBJECTIVE QUESTIONS)

1. The domain of the function $f(x)=\dfrac{\sqrt{-\log_{0.3}(x-1)}}{\sqrt{x^2+2x+8}}$ is

(A) (1, 4) (B) (–2, 4) (C) (2, 4) (D) [2, ∞)

Sol.

2. The domain of the function

$f(x) = \log_{1/2}\left(-\log_2\left(1+\dfrac{1}{\sqrt[4]{x}}\right)-1\right)$ is

(A) $0 < x < 1$ (B) $0 < x \le 1$ (C) $x \ge 1$ (D) null set

Sol.

3. If $q^2 - 4pr = 0$, $p > 0$, then the domain of the function, $f(x) = \log(px^3 + (p + q)x^2 + (q + r)x + r)$ is

(A) $R - \left\{-\dfrac{q}{2p}\right\}$ (B) $R - \left[(-\infty,-1] \cup \left\{-\dfrac{q}{2p}\right\}\right]$

(C) $R-\left[(-\infty,-1] \cap \left\{-\dfrac{q}{2p}\right\}\right]$ (D) none of these

Sol.

4. If domain of f(x) is (– ∞, 0] then domain of $f(6\{x\}^2 - 5\{x\} + 1)$ is
(where {∗} represetns fractional part function)

(A) $\bigcup_{n\in I}\left[n+\dfrac{1}{3},n+\dfrac{1}{2}\right]$ (B) (–∞, 0)

(C) $\bigcup_{n\in I}\left[n+\dfrac{1}{6}n+1\right]$ (D) None of these

Sol.

5. Find domain of the function

$f(x) = \sqrt{-\log_{\frac{x+4}{2}}\left(\log_2\dfrac{2x-1}{3+x}\right)}$

(A) (–4, –3) ∪ (4, ∞) (B) (– ∞, –3) ∪ (4, ∞)
(C) (– ∞, – 4) ∪ (3, ∞) (D) None of these

Sol.

6. The domain of the function $\sqrt{\log_{1/3}\log_4([x]^2-5)}$ is (where [x] denotes greatest integer function)

(A) $[-3, -2) \cup [3, 4)$ (B) $[-3, -2) \cup (2, 3]$

(C) $R - [-2, 3)$ (D) $R - [-3, 3]$

Sol.

7. Let $f(x) = (x^{12} - x^9 + x^4 - x + 1)^{-1/2}$. the domain of the function is

(A) $(1, +\infty)$ (B) $(-\infty, -1)$ (C) $(-1, 1)$ (D) $(-\infty, \infty)$

Sol.

8. Range of $f(x) = 4^x + 2^x + 1$ is

(A) $(0, \infty)$ (B) $(1, \infty)$ (C) $(2, \infty)$ (D) $(3, \infty)$

Sol.

9. Range of f(x) = $\log_{\sqrt{5}}$ {$\sqrt{2}$ (sin x –cos x) + 3} is

(A) [0, 1] (B) [0, 2] (C) $\left[0, \frac{3}{2}\right]$ (D) None of these

Sol.

10. The range of the functin

f(x) = $\log_{\sqrt{2}}$ (2– $\log_2$ (16 $\sin^2$ x + 1)) is

(A) (–∞, 1) (B) (–∞, 2) (C) (–∞, 1] (D) (–∞, 2]

Sol.

11. If [2 cos x] + [sin x] = –3, then the range of the function, f(x) = sin x + $\sqrt{3}$ cos x in [0, 2π] is

(where [∗] dentoes greatest integer function)

(A) [–2, –1) (B) (–2, –1]

(C) (–2, –1) (D) [–2, – $\sqrt{3}$)

Sol.

12. The range of the function f(x) = ${}^{7-x}P_{x-3}$ is

(A) {1, 2, 3} (B) {1, 2, 3, 4, 5, 6}

(C) {1, 2, 3, 4} (D) {1, 2, 3, 4, 5}

Sol.

13. Range of the function f(x)= $\begin{vmatrix} \cos\frac{x}{2} & 1 & 1 \\ 1 & \cos\frac{x}{2} & -\cos\frac{x}{2} \\ -\cos\frac{x}{2} & 1 & -1 \end{vmatrix}$ is

(A) [0, 2] (B) [0, 4] (C) [2, 4] (D) [1, 3]

Sol.

14. In the square ABCD with side AB = 2, two points M & N are on the adjacent sides of the square such that MN is parallel to the diagonal BD. If x is the distance of MN from the vertex A and f(x) = Area (ΔAMN), then range of f(x) is

(A) $(0,\sqrt{2}]$ (B) (0, 2] (C) $(0,2\sqrt{2}]$ (D) $(0,2\sqrt{3}]$

Sol.

15. Let f be a real valued function defined by

$f(x) = \dfrac{e^x - e^{-|x|}}{e^x + e^{|x|}}$ then the range of f(x) is

(A) R (B) [0, 1] (C) [0, 1) (D) $\left[0,\frac{1}{2}\right)$

Sol.

16. If $f(x) = \dfrac{\sin^2 x + 4\sin x + 5}{2\sin^2 x + 8\sin x + 8}$, then range of f(x) is

(A) $\left(\frac{1}{2},\infty\right)$ (B) $\left(\frac{5}{9},1\right)$ (C) $\left[\frac{5}{9},1\right]$ (D) $\left[\frac{5}{9},\infty\right)$

Sol.

17. The number of solution(s) of the equation
$[x] + 2\{-x\} = 3x$, is/are
(where [$*$] represents the greatest integer function and { $*$ } denotes the fractional part of x)
(A) 1 (B) 2 (C) 3 (D) 0
Sol.

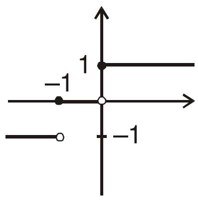
18. The number of solutions of the equation
$[\sin^{-1} x] = x - [x]$ is
(where [$*$] denotes the greatest integer function)
(A) 0 (B) 1 (C) 2 (D) infinitely many
Sol.

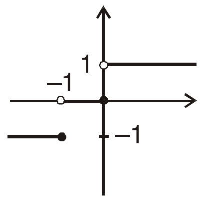
19. The sum

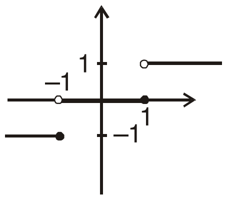
$$\left[\frac{1}{2}\right]+\left[\frac{1}{2}+\frac{1}{2000}\right]+\left[\frac{1}{2}+\frac{2}{2000}\right]+\left[\frac{1}{2}+\frac{3}{2000}\right]+\ldots\ldots+\left[\frac{1}{2}+\frac{1999}{2000}\right]$$

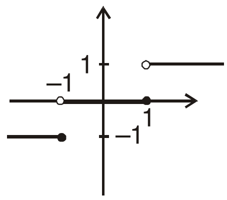
is equal to
(where [$*$] denotes the greatest integer function)
(A) 1000 (B) 999 (C) 1001 (D) None of these
Sol.

20. Which of the following represents the graph of $f(x) = \text{sgn}([x + 1])$

(A)


(B)


(C)


(D)


Sol.

21. If $f(x)=2\sin^2\theta+4\cos(x+\theta)\sin x.\sin\theta+\cos(2x+2\theta)$

then value of $f^2(x) + f^2\left(\frac{\pi}{4}-x\right)$ is

(A) 0 (B) 1 (C) –1 (D) x^2

Sol.

22. If A, B, C are three decimal numbers and p = [A + B + C] and q = [A] + [B] + [C] then maximum value of p – q is (where [*] represents greatest integer function).

(A) 0 (B) 1 (C) 2 (D) 3

Sol.

23. Let $f(x) = ax^2 + bx + c$, where a, b, c are rational and $f : Z \to Z$, where Z is the set of integers. Then a + b is

(A) a negative integer (B) an integer

(C) non-integral rational number (D) None of these

Sol.

24. Which one of the following pair of functions are identical ?

(A) $e^{(\ln x)/2}$ and $\sqrt{x}$

(B) $\tan^{-1}(\tan x)$ & $\cot^{-1}(\cot x)$

(C) $\cos^2 x + \sin^4 x$ and $\sin^2 x + \cos^4 x$

(D) $\frac{|x|}{x}$ and sgn (x) where sgn(x) stands for signum function.

Sol.

25. The function $f : [2, \infty) \to Y$ defined by $f(x) = x^2 - 4x + 5$ is both one–one & onto if

(A) Y = R (B) Y = $[1, \infty)$ (C) Y = $[4, \infty)$ (D) Y = $[5, \infty)$

Sol.

26. Let $f : R \to R$ be a function defined by

$$f(x) = \frac{2x^2 - x + 5}{7x^2 + 2x + 10}$$ then f is

(A) one – one but not onto

(B) onto but not one – one

(C) onto as well as one – one

(D) neither onto nor one – one

Sol.

27. Let $f : R \to R$ be a function defined by $f(x) = x^3 + x^2 + 3x + \sin x$. Then f is

(A) one – one & onto (B) one – one & into

(C) many one & onto (D) many one & into

Sol.

28. If $f(x) = \frac{4a-7}{3}x^3 + (a-3)x^2 + x + 5$ is a one–one function, then

(A) $2 \le a \le 8$ (B) $1 \le a \le 2$
(C) $0 \le a \le 1$ (D) None of these

Sol.

29. Let f: $(e, \infty) \to R$ be defined by $f(x) = \ln(\ln(\ln x))$, then

(A) f is one one but not onto
(B) f is onto but not one – one
(C) f is one–one and onto
(D) f is neither one–one nor onto

Sol.

30. If $f(x) = 2[x] + \cos x$, then f: $R \to R$ is
(where [$*$] denotes greatest integer function)

(A) one–one and onto (B) one–one and into
(C) many–one and into (D) many–one and onto

Sol.

31. If f : $R \to S$, defined by $f(x) = \sin x - \sqrt{3}\cos x + 1$, is onto, then the interval of S is

(A) [0, 3] (B) [–1, 1] (C) [0, 1] (D) [–1, 3]

Sol.

32. The function f : $R \to R$ defined by $f(x) = 6^x + 6^{|x|}$ is

(A) one-one and onto (B) many-one and onto
(C) one-one and into (D) many-one and into

Sol.

33. If the real-valued function f(x) = px + sinx is a bijective function, then the set of all possible values of p $\in$ R is
(A) R – {0} (B) R (C) $(0, \infty)$ (D) None of these
Sol.

34. Let S be the set of all triangles and R^+ be the set of positive real numbers. Then the function, $f : S \to R^+$, $f(\Delta)$ = area of the Δ, where $\Delta \in S$ is
(A) injective but not surjective
(B) surjective but not injective
(C) injective as well as surjective
(D) neither injective nor surjective
Sol.

35. Let 'f' be a function from R to R given by
$f(x) = \dfrac{x^2-4}{x^2+1}$. Then f(x) is
(A) one-one and into (B) one-one and onto
(C) many-one and into (D) many-one and onto
Sol.

36. Function $f : (-\infty, 1) \to (0, e^5]$ defined by
$f(x) = e^{-(x^2-3x+2)}$ is
(A) many one and onto (B) many one and into
(C) one one and onto (D) one one and into
Sol.

37. If $f(x) = \cot^{-1} x : R^+ \to \left(0, \dfrac{\pi}{2}\right)$
and $g(x) = 2x - x^2 : R \to R$. Then the range of the function f(g(x)) wherever define is
(A) $\left(0, \dfrac{\pi}{2}\right)$ (B) $\left(0, \dfrac{\pi}{4}\right)$ (C) $\left[\dfrac{\pi}{4}, \dfrac{\pi}{2}\right)$ (D) $\left(\dfrac{\pi}{4}\right)$
Sol.

38. $f(x) = |x - 1|$, $f : R^+ \to R$; $g(x) = e^x$, $g : [-1, \infty) \to R$
If the function fog(x) is defined, then its domain and range respectively are
(A) $(0, \infty)$ & $[0, \infty)$ (B) $[-1, \infty)$ & $[0, \infty)$
(C) $[-1, \infty)$ & $\left(1 - \dfrac{1}{e}, \infty\right]$ (D) $[-1, \infty)$ & $\left(\dfrac{1}{e} - 1, \infty\right]$

Sol.

39. Let g(x) = 1 + x – [x] and f(x) = $\begin{cases} -1 & \text{if } x < 0 \\ 0 & \text{if } x = 0 \\ 1 & \text{if } x > 0 \end{cases}$,

then $\forall$ x, fog(x) equals
(where [$*$] represents greatest integer function).
(A) x (B) 1 (C) f(x) (D) g(x)
Sol.

40. Let f: [0, 1] $\rightarrow$ [1, 2] defined as f(x) = 1 + x and g : [1, 2] $\rightarrow$ [0, 1] defined as g(x) = 2 – x then the composite function gof is
(A) injective as well as surjective
(B) Surjective but not injective
(C) Injective but non surjective
(D) Neither injective nor surjective
Sol.

41. Let f & g be two functions both being defined from R $\rightarrow$ R as follows f(x) = $\frac{x+|x|}{2}$ and

g(x) = $\begin{cases} x & \text{for } x < 0 \\ x^2 & \text{for } x \geq 0 \end{cases}$. Then

(A) fog is defined but gof is not
(B) gof is defined but fog is not
(C) both fog & gof are defined but they are unequal
(D) both gof & fog are defined and they are equal
Sol.

42. If y = f (x) satisfies the condition

$f\left(x+\frac{1}{x}\right) = x^2 + \frac{1}{x^2}$ $(x \neq 0)$ then f(x) equals

(A) $-x^2 + 2$ (B) $-x^2 - 2$ (C) $x^2 + 2$ (D) $x^2 - 2$
Sol.

43. If f(1) = 1 and f(n + 1) = 2f(n) + 1 if n $\geq$ 1, then f(n) is equal to
(A) $2^n + 1$ (B) 2^n (C) $2^n - 1$ (D) $2^{n-1} - 1$
Sol.

44. A function $f : R \to R$ satisfies the condition,
$x^2 f(x) + f(1 - x) = 2x - x^4$. Then f(x) is
(A) $-x^2 - 1$ (B) $-x^2 + 1$ (C) $x^2 - 1$ (D) $-x^4 + 1$
Sol.

45. A real valued function f(x) satisfies the functional equation $f(x - y) = f(x) f(y) - f(a - x) f(a + y)$ where a is a given constant and $f(0) = 1$, $f(2a - x)$ is equal to
(A) $f(-x)$ (B) $f(a) + f(a - x)$ (C) $f(x)$ (D) $-f(x)$
Sol.

46. If $f : R \to R$ satisfies $f(x + y) = f(x) + f(y)$, for all $x, y \to R$ and $f(1) = 7$, then $\sum_{r=1}^{n} f(r)$ is
(A) $\frac{7n}{2}$ (B) $\frac{7(n+1)}{2}$ (C) $7n(n+1)$ (D) $\frac{7n(n+1)}{2}$.
Sol.

47. The function $f(x) = \log\left(\frac{1+\sin x}{1-\sin x}\right)$ is
(A) even (B) odd
(C) neither even nor odd (D) both even & odd
Sol.

48. If the graph of the function $f(x) = \frac{a^x - 1}{x^n(a^x + 1)}$ is symmetric about y–axis, then n is equal to
(A) 2 (B) 2/3 (C) 1/4 (D) –1/3
Sol.

49. The graph of the function $y = f(x)$ is symmetrical about the line $x = 2$, then
(A) $f(x + 2) = f(x - 2)$ (B) $f(2 + x) = f(2 - x)$
(C) $f(x) = f(-x)$ (D) $f(x) = -f(-x)$

Sol.

50. If $f(-x) = -f(x)$, then $f(x)$ is
(A) neither odd nor even (B) an odd function
(C) an even function (D) periodic function
Sol.

51. If $g : [-2, 2] \rightarrow R$ where $g(x)=x^3+\tan x + \left[\frac{x^2+1}{p}\right]$ be an odd function , then the value of the parameter P is
(A) $-5 < P < 5$ (B) $P < 5$ (C) $P>5$ (D) None of these
Sol.

52. It is given that f(x) is an even function and satisfy the relation $f(x) = \dfrac{xf(x^2)}{2+\tan^2 x.f(x^2)}$ then the value of f(10) is
(A) 10 (B) 100 (C) 50 (D) None of these

Sol.

53. Fundamental period of $f(x) = \sec(\sin x)$ is
(A) $\pi/2$ (B) 2π (C) π (D) a periodic
Sol.

54. If $f(x) = \sin\sqrt{[a]}\,x$ has π as its fundamental period then (where [$*$] denotes the gratest integer function)
(A) a = 1 (B) a = 9 (C) $a \in [1, 2)$ (D) $a \in [4, 5)$
Sol.

55. The fundamental period of the function,
$f(x) = x + a - [x + b] + \sin \pi x + \cos 2\pi x + \sin 3\pi x + \cos 4\pi x + \ldots.. + \sin(2n-1)\pi x + \cos 2n\pi x$
or every $a, b \in R$ is
(where [$*$] denotes the greatest integer function)
(A) 2 (B) 4 (C) 1 (D) 0
Sol.

56. The period of $\sin\frac{\pi}{4}[x] + \cos\frac{\pi x}{2} + \cos\frac{\pi}{3}[x]$, where [x] denotes the integral part of x is
(A) 8 (B) 12 (C) 24 (D) Non–periodic

Sol.

57. The fundamental period of function

$f(x) = [x] + \left[x+\frac{1}{3}\right]+\left[x+\frac{2}{3}\right] - 3x + 15$

(A) 1/3 (B) 2/3 (C) 1 (D) Non–periodic

Sol.

58. Which one of the following is true.

(A) $f(x) = \frac{16^x - 1}{4^x}$ is an odd function

(B) f(x) = sin |x| is an odd function

(C) if sin x + cos a x is periodic then 'a' is irrational

(D) if f_1 (x), f_2 (x) are periodic then their sum function will always be periodic

Sol.

59. Let f(x) = x (2 – x), $0 \le x \le 2$. If the definition of 'f' is extended over the set, R – [0, 2] by f(x + 2) = f(x), then 'f' is a

(A) periodic function of period 1

(B) non-periodic function

(C) periodic function of period 2

(D) periodic function of period 1/2

Sol.

60. Let f(2, 4) $\rightarrow$ (1, 3) be a function defined by

$f(x) = x - \left[\frac{x}{2}\right]$, then f^{-1} (x) is equal to

(where [$*$] denotes the greatest integer function)

(A) 2x (B) $x + \left[\frac{x}{2}\right]$ (C) x + 1 (D) x – 1

Sol.

61. The mapping f : R $\rightarrow$ R given by

$f(x) = x^3 + ax^2 + bx + c$ is a bijection if

(A) $b^2 \le 3a$ (B) $a^2 \le 3b$ (C) $a^2 \ge 3b$ (D) $b^2 \ge 3a$

Sol.

EXERCISE – II MULTIPLE CORRECT (OBJECTIVE QUESTIONS)

1. Let f : [–1, 1] $\rightarrow$ [0, 2] be a linear function which is onto then f(x) is/are

(A) 1 – x (B) 1 + x (C) x – 1 (D) x + 2

Sol.

2. In the following functions defined from [–1, 1] to [–1, 1] the functions which are not bijective are

(A) $\sin(\sin^{-1} x)$ (B) $\frac{2}{\pi}\sin^{-1}(\sin x)$

(C) $(\text{sgn } x)\, \ell n\, e^x$ (D) $x^3 \text{ sgn } x$

Sol.

3. A function 'f' from the set of natural numbers to integers defined by $f(n)=\begin{cases} \frac{n-1}{2}, & \text{when n is odd} \\ -\frac{n}{2}, & \text{when n is even} \end{cases}$ is.

(A) one–one (B) many–one (C) onto (D) into

Sol.

4. Let $f(x) = \left(\frac{1-x}{1+x}\right)$, $0 \le x \le 1$ and $g(x) = 4x(1 - x)$, $0 \le x \le 1$. then

(A) $fog = \frac{1-4x+4x^2}{1+4x-4x^2}$, $0 \le x \le 1$

(B) $fog = \frac{1-4x-4x^2}{1+4x-4x^2}, \frac{1}{2} \le x \le 1$

(C) $gof = \frac{8x(1-x)}{(1+x)^2}$, $0 \le x \le 1$

(D) $gof = \frac{8x(1+x)}{(1+x)^2}$, $0 \le x \le 1$

Sol.

5. If 'f' and 'g' are bijective functions and gof is defined then gof must be
(A) injective (B) surjective
(C) bijective (D) into only
Sol.

6. The period of the function $f(x) = \sin^4 3x + \cos^4 3x$ is
(A) $\pi/6$ (B) $\pi/3$ (C) $\pi/2$ (D) $\pi/12$
Sol.

7. If $f : R \to [-1, 1]$, where $f(x) = \sin \pi/2\ [x]$, (where $[*]$ dentoes the greatest integer function) then
(A) f(x) is onto (B) f(x) is into
(C) f(x) is periodic (D) f(x) is many one
Sol.

8. If $F(x) = \dfrac{\sin \pi[x]}{\{x\}}$, then F(x) is
(where $\{*\}$ denotes fractional part of function and $[*]$ denotes greatest integer function)
(A) periodic with fundamental period 1
(B) even
(C) range is singleton
(D) identical to $\text{sgn}\left(\text{sgn}\dfrac{\{x\}}{\sqrt{\{x\}}}\right) - 1$,
(where $\{ * \}$ denotes fractional part of function and $[*]$ denotes greatest integer function and sgn (x) is a signum function)
Sol.

9. Function $f(x) = \sin x + \tan x + \text{sgn}(x^2 - 6x + 10)$ is
(A) periodic with period 2π
(B) periodic with period π
(C) Non–periodic
(D) periodic with period 4π
Sol.

10. Which of the following functions are periodic ?
(A) $f(x) = \text{sgn}(e^{-x})$

(B) $f(x) = \begin{cases} 1 \text{ if } x \text{ is a rational number} \\ 0 \text{ if } x \text{ is an irrational number} \end{cases}$

(C) $f(x) = \sqrt{\dfrac{8}{1+\cos x} + \dfrac{8}{1-\cos x}}$

(D) $f(x) = \left[x + \dfrac{1}{2}\right] + \left[x - \dfrac{1}{2}\right] + 2[-x]$
(where [$*$] denotes greatest integer function)
Sol.

EXERCISE – III

SUBJECTIVE QUESTIONS

1. Find the domain of each of the following functions

(i) $f(x) = \dfrac{x^3 - 5x + 3}{x^2 - 1}$

Sol.

(ii) $f(x) = \dfrac{1}{\sqrt{x + |x|}}$

Sol.

(iii) $f(x) = e^{x + \sin x}$

Sol.

(iv) $f(x) = \dfrac{1}{\log_{10}(1 - x)} + \sqrt{x + 2}$

Sol.

(v) $\log_x \log_2 \left(\dfrac{1}{x - 1/2}\right)$

Sol.

(vi) $f(x) = \sqrt{3 - 2^x - 2^{1-x}}$

Sol.

(vii) $f(x) = \sqrt{1 - \sqrt{1 - x^2}}$

Sol.

(viii) $f(x) = (x^2 + x + 1)^{-3/2}$

Sol.

(ix) $f(x) = \sqrt{\frac{x-2}{x+2}} + \sqrt{\frac{1-x}{1+x}}$

Sol.

(x) $f(x) = \sqrt{\tan x - \tan^2 x}$

Sol.

(xi) $f(x) = \frac{1}{\sqrt{1-\cos x}}$

Sol.

(xii) $f(x) = \sqrt{\log_{1/4}\left(\frac{5x - x^2}{4}\right)}$

Sol.

(xiii) $f(x) = \log_{10}(1 - \log_{10}(x^2 - 5x + 16))$

Sol.

2. Find the range of each of the following functions

(i) $f(x) = |x - 3|$

Sol.

(ii) $f(x) = \frac{x}{1+x^2}$

Sol.

(iii) $f(x) = \sqrt{16 - x^2}$

Sol.

(iv) $f(x) = \dfrac{|x-4|}{x-4}$

Sol.

(v) $f(x) = 5 + 3 \sin x + 4 \cos x$

Sol.

(vi) $f(x) = \dfrac{1}{1+\sqrt{x}}$

Sol.

(vii) $f(x) = 2 - 3x - 5x^2$

Sol.

(viii) $3 |\sin x| - 4 |\cos x|$

Sol.

(ix) $\dfrac{\sin x}{\sqrt{1+\tan^2 x}} + \dfrac{\cos x}{\sqrt{1+\cot^2 x}}$

Sol.

(x) $f(x) = 1 - |x - 2|$

Sol.

(xi) $f(x) = \dfrac{1}{\sqrt{x-5}}$

Sol.

(xii) f(x) = $\frac{1}{2-\cos 3x}$

Sol.

(xiii) f(x) = $\frac{x+2}{x^2-8x-4}$

Sol.

(xiv) f(x) = $\frac{x^2-2x+4}{x^2+2x+4}$

Sol.

(xv) f(x) = 3 sin $\sqrt{\frac{\pi^2}{16}-x^2}$

Sol.

(xvi) f(x) = $x^4 - 2x^2 + 5$

Sol.

(xvii) f(x) = $x^3 - 12x$, where $x \in [-3, 1]$

Sol.

(xviii) f(x) = $\sin^2 x + \cos^4 x$

Sol.

3. Find the domain and the range of each of the following functions

(i) f(x) = $\frac{1}{\sqrt{4+3\sin x}}$

Sol.

(ii) $f(x) = x!$
Sol.

(iii) $f(x) = \dfrac{x^2-9}{x-3}$

Sol.

(iv) $f(x) = \sin^2(x^3) + \cos^2(x^3)$
Sol.

4. If $f(x) = \dfrac{4^x}{4^x+2}$, then show that $f(x) + f(1 - x) = 1$

Sol.

5. Draw the graph of the function $f(x)=|x^2-4|x|+3|$ and also find the set of values of 'a' for which the equation $f(x) = a$ has exactly four distinct real roots.

Sol.

6. Solve the following equation for $x : 2x + 3[x] - 4\{-x\} = 4$
(where $[\ast]$ & $\{\ast\}$ denotes integral and fractional part of x)
Sol.

7. Let f(x) be defined on [−2, 2] and is given by

$f(x) = \begin{cases} -1 & , \quad -2 \le x \le 0 \\ x-1 & , \quad 0 \le x \le 2 \end{cases}$ and $g(x) = f(|x|) + |f(x)|$,

then find g(x).
Sol.

8. Check whether following pairs of functions are identical or not ?

(i) $f(x) = \sqrt{x^2}$ & $g(x) = (\sqrt{x})^2$

Sol.

(ii) $f(x) = \sec(\sec^{-1} x)$ & $g(x) = \text{cosec}(\text{cosec}^{-1} x)$

Sol.

(iii) $f(x) = \sqrt{\frac{1+\cos 2x}{2}}$ & $g(x) = \cos x$

Sol.

(iv) $f(x) = x$ and $g(x) = e^{\ell nx}$

Sol.

9. Find whether the following functions are one–one or many–one

(i) $f(x) = |x^2 + 5x + 6|$

Sol.

(ii) $f(x) = |\log x|$

Sol.

(iii) $f(x) = \sin 4x, x \in \left(-\frac{\pi}{8}, \frac{\pi}{8}\right)$

Sol.

(iv) $f(x) = x + \frac{1}{x}, x \in (0, \infty)$

Sol.

(v) $f(x) = \sqrt{1-e^{\left(\frac{1}{x}-1\right)}}$

Sol.

(vi) $f(x) = \dfrac{3x^2}{4\pi} - \cos \pi x$

Sol.

10. Let $f : D \to R$ where D is its domain. Find whether the following functions are into/onto.

(i) $f(x) = \dfrac{1+x^6}{x^3}$

Sol.

(ii) $f(x) = x \cos x$

Sol.

(iii) $f(x) = \dfrac{1}{\sin\sqrt{|x|}}$

Sol.

(iv) $\tan(2 \sin x)$

Sol.

11. Classify the following functions f(x) defined in $R \to R$ as injective , surjective, both or none.

(i) $f(x) = x\,|\,x\,|$

Sol.

(ii) $f(x) = x^2$

Sol.

(iii) $f(x) = \frac{x^2}{1+x^2}$

Sol.

(iv) $f(x) = x^3 - 6x^2 + 11x - 6$

Sol.

12. Let $f : A \to A$ where $A = \{x : -1 \le x \le 1\}$. Find whether the following function are bijective.

(i) $x - \sin x$

Sol.

(ii) $x\,|x|$

Sol.

(iii) $\tan \frac{\pi x}{4}$

Sol.

(iv) x^4

Sol.

13. Find fog and gof, if

(i) $f(x) = e^x$; $g(x) = \log x$

Sol.

(ii) $f(x) = |x|$; $g(x) = \sin x$

Sol.

(iii) $f(x) = \sin^{-1}x$; $g(x) = x^2$

Sol.

(iv) $f(x) = x^2 + 2$; $g(x) = 1 - \frac{1}{1-x}$, $x \ne 1$

Sol.

14. If $f(x) = \begin{cases} 1+x^2 & x \le 1 \\ x+1 & 1 < x \le 2 \end{cases}$ and $g(x) = 1 - x$; $-2 \le x \le 1$ then define function fog(x).

Sol.

15. Let f(x) = $\begin{cases} 1+x, & 0 \le x \le 2 \\ 3-x, & 2 < x \le 3 \end{cases}$. Find fof.

Sol.

16. If f(x) = $\ln(x^2 - x+2)$; $R^+ \to R$ and g(x) = {x}+1 ; [1, 2] $\to$ [1, 2],
(where { $*$ } denotes fractional part of x). Find the domain and range of f(g(x)) when defined.

Sol.

17. Let f(x) be a polynomial function satisfying the relation $f(x) \,.\, f\left(\frac{1}{x}\right) = f(x) + f\left(\frac{1}{x}\right)$ $\forall$ $x \in R - \{0\}$ and f(3) = −26. Determine f′(1).

Sol.

18. If f(x + y) = f(x) . f(y) $\forall$ x, y $\in$ N and f(1) = 2, then find $\sum_{n=1}^{10} f(n)$.

Sol.

19. If f(x) = $\begin{cases} x^2 + \sin x & 0 \le x < 1 \\ x + e^{-x} & x \ge 1 \end{cases}$ then extend the definition of f(x) for x $\in$ (− ∞, 0) such that f(x) becomes

(i) An even function

Sol.

(ii) An odd function

Sol.

20. Examine whether the following functions are even or odd or none.

(i) f(x) = $\frac{(1+2^x)^7}{2^x}$

Sol.

(ii) f(x) = $\frac{\sec x + x^2 - 9}{x \sin x}$

Sol.

(iii) f(x) = $\sqrt{1+x+x^2} - \sqrt{1-x+x^2}$

Sol.

(iv) f(x) = $\begin{cases} x\,|x| & , \quad x \le -1 \\ [1+x]-[x-1] & , \quad -1 < x < 1 \\ -x\,|x| & , \quad x \ge 1 \end{cases}$

Sol.

(v) f(x) = $\dfrac{2x(\sin x + \tan x)}{2\left[\dfrac{x+2\pi}{\pi}\right]-3}$,

where [∗] denotes greatest integer function.

Sol.

21. Find the period of the following functions (where [∗] denotes greatest integer function)

(i) f(x) = 2 + 3 cos (x – 2)

Sol.

(ii) f(x) = sin 3x + $\cos^2$x + |tan x|

Sol.

(iii) f(x) = sin $\dfrac{\pi x}{4}$ + sin $\dfrac{\pi x}{3}$

Sol.

(iv) f(x) = cos $\dfrac{3}{5}$x – sin $\dfrac{2}{7}$ x.

Sol.

(v) f(x) = [sin 3x] + |cos 6x|

Sol.

(vi) $f(x) = \dfrac{1}{1-\cos x}$

Sol.

(vii) $f(x) = \dfrac{\sin 12x}{1+\cos^2 6x}$

Sol.

(viii) $f(x) = \sec^2 x + \text{cosec}^3 x$

Sol.

22. Find the period of the following functions.

(i) $f(x) = 1 - \dfrac{\sin^2 x}{1+\cot x} - \dfrac{\cos^2 x}{1+\tan x}$

Sol.

(ii) $f(x) = \log(2 + \cos 3x)$

Sol.

(iii) $f(x) = \tan \dfrac{\pi}{2}[x]$,

where [∗] denotes greatest integer function

Sol.

(iv) $f(x)\ e^{\ln \sin x} + \tan^3 x - \text{cosec}(3x - 5)$

Sol.

(v) $f(x) = \dfrac{1}{2}\left(\dfrac{|\sin x|}{\cos x} + \dfrac{\sin x}{|\cos x|}\right)$

Sol.

(vi) $f(x) = \sin x + \tan \dfrac{x}{2} + \sin \dfrac{x}{2^2} + \tan \dfrac{x}{2^3} + \dots\dots\dots$

$+ \sin \dfrac{x}{2^{n-1}} + \tan \dfrac{x}{2^n}$

Sol.

(vii) $f(x) = \dfrac{\sin x + \sin 3x}{\cos x + \cos 3x}$

Sol.

23. Find the period of f(x) satisfying the condition

(i) $f(x + p) = 1 + \{1 - 3f(x) + 3f^2(x) - f^3(x)\}^{1/3}$

Sol.

(ii) $f(x - 1) + f(x + 3) = f(x + 1) + f(x + 5)$

Sol.

24. Let $f : R \to R$ be defined by $f(x) = \dfrac{e^{2x} - e^{-2x}}{2}$. Is f(x) invertible ? If yes, then find its inverse.

Sol.

25. Let $f : \left[-\dfrac{\pi}{3}, \dfrac{\pi}{6}\right] \to B$ defined by

$f(x) = 2\cos^2 x + \sqrt{3}\sin 2x + 1$. Find B such that f^{-1} exists. Also find $f^{-1}(x)$.

Sol.

26. Let $f : N \to N$ where $f(x) = x + (-1)^{x-1}$ find the inverse of f.

Sol.

EXERCISE – IV ADVANCED SUBJECTIVE QUESTIONS

1. Find the domain of definitions of the functions (Read the symbols [*] and {*} as greatest integers and fractional part functions respectively.)

(i) f(x) = $\sqrt{\cos 2x} + \sqrt{16 - x^2}$

Sol.

(ii) f(x) = $\log_7 \log_5 \log_3 \log_2 (2x^3 + 5x^2 - 14x)$

Sol.

(iii) f(x) = $ln\left(\sqrt{x^2 - 5x - 24} - x - 2\right)$

Sol.

(iv) f(x) = $\sqrt{\dfrac{1-5^x}{7^{-x}-7}}$

Sol.

(v) y = $\log_{10} \sin (x - 3) + \sqrt{16 - x^2}$

Sol.

(vi) f(x) = $\log_{100x}\left(\dfrac{2\log_{10} x + 1}{-x}\right)$

Sol.

(vii) $f(x) = \dfrac{1}{\sqrt{4x^2 - 1}} + \ln x(x^2 - 1)$

Sol.

(viii) $f(x) = \sqrt{\log_{\frac{1}{2}} \dfrac{x}{x^2 - 1}}$

Sol.

(ix) $f(x) = \sqrt{x^2 - |x|} + \dfrac{1}{\sqrt{9 - x^2}}$

Sol.

(x) $f(x) = \sqrt{(x^2 - 3x - 10).\ell n^2(x - 3)}$

Sol.

(xi) $f(x) = \sqrt{\log_x(\cos 2\pi x)}$

Sol.

(xii) $f(x) = \dfrac{\sqrt{\cos x - \frac{1}{2}}}{\sqrt{6 + 35x - 6x^2}}$

Sol.

(xiii) $f(x) = \sqrt{\log_{1/3}(\log_4([x]^2 - 5))}$

Sol.

(xiv) $f(x) = \dfrac{1}{[x]} + \log_{(2\{x\}-5)}(x^2 - 3x + 10) + \dfrac{1}{\sqrt{1-|x|}}$

Sol.

(xv) $f(x) = \log_x \sin x$

Sol.

(xvi) $f(x) = \log_2\left(-\log_{1/2}\left(1 + \dfrac{1}{\sin\left(\frac{x^\circ}{100}\right)}\right)\right) + \sqrt{\log_{10}(\log_{10} x) - \log_{10}(4 - \log_{10} x) - \log_{10} 3}$

Sol.

(xvii) $f(x) = \dfrac{1}{[x]} + \log_{1-\{x\}}(x^2-3x+10) + \dfrac{1}{\sqrt{2-|x|}} + \dfrac{1}{\sqrt{\sec(\sin x)}}$

Sol.

(xviii) $f(x) = \sqrt{(5x-6-x^2)[\{\ell n\{x\}\}]}$ +

$\sqrt{(7x-5-2x^2)} + \left(\ell n\left(\dfrac{7}{2}-x\right)\right)^{-1}$

Sol.

(xix) If $f(x) = \sqrt{x^2-5x+4}$ & $g(x) = x + 3$, then find the domain of $\dfrac{f}{g}$ (x).

Sol.

2. Find the domain & range of the following functions .

(i) $y = \log_{\sqrt{5}}\left(\sqrt{2}(\sin x - \cos x) + 3\right)$

Sol.

(ii) $y = \dfrac{2x}{1+x^2}$

Sol.

(iii) $f(x) = \dfrac{x^2-3x+2}{x^2+x-6}$

Sol.

(iv) $f(x) = \dfrac{x}{1+|x|}$

Sol.

(v) $y = \sqrt{2-x} + \sqrt{1+x}$

Sol.

(vi) $f(x) = \log_{(\text{cosec } x - 1)}(2 - [\sin x] - [\sin x]^2)$

Sol.

(vii) $f(x) = \dfrac{\sqrt{x+4} - 3}{x-5}$

Sol.

3. Find the set of real value(s) of p for which the equation $|2x + 5| + |2x - 5| = px + 10$ has two solutions.

Sol.

4. A function f defined for all real numbers is defined as follows for $x \geq 0$: $f(x) = \begin{cases} x, & 0 \leq x \leq 1 \\ 1, & x > 1 \end{cases}$

How is f defined for $x \leq 0$ if :

(i) f is even **(ii)** f is odd ?

Sol.

5. The function f(x) is defined on the interval [0,1]. Find the domain of definition of the functions.

(a) f (sin x) **(b)** f (2x+3)

Sol.

6. A function $f : \left[\frac{1}{2}, \infty\right) \to \left[\frac{3}{4}, \infty\right)$ defined as, $f(x) = x^2 - x + 1$. Then solve the equation $f(x) = f^{-1}(x)$.

Sol.

7. Compute the inverse of the functions

(i) $f(x) = \ln\left(x + \sqrt{x^2 + 1}\right)$

Sol.

(ii) $f(x) = 2^{\frac{x}{x-1}}$

Sol.

(iii) $y = \dfrac{10^x - 10^{-x}}{10^x + 10^{-x}}$

Sol.

8. Write explicitly, functions of y defined by the following equations and also find the domains of definition of the given implicit functions

(i) $10^x + 10^y = 10$

Sol.

(ii) $x + |y| = 2y$

Sol.

9. Find whether the following functions are even or odd or none

(a) $f(x) = \log\left(x+\sqrt{1+x^2}\right)$

Sol.

(b) $f(x) = \dfrac{x(a^x+1)}{a^x-1}$

Sol.

(c) $f(x) = \sin x + \cos x$

Sol.

(d) $f(x) = x\sin^2 x - x^3$

Sol.

(e) $f(x)= \sin x - \cos x$

Sol.

(f) $f(x) = \dfrac{(1+2^x)^2}{2^x}$

Sol.

(g) $f(x)= \dfrac{x}{e^x-1}+\dfrac{x}{2}+1$

Sol.

(h) $f(x) = [(x+1)^2]^{1/3} + [(x-1)^2]^{1/3}$

Sol.

10. Solve the following problems from (i) to (v) on functional equation.

(i) The function f(x) defined on the real numbers has the property that $f(f(x)) \cdot (1 + f(x)) = -f(x)$ for all x in the domain of f. If the number 3 is the domain and range of f, compute the value of f(3).

Sol.

(ii) Suppose f is a real function satisfying $f(x + f(x)) = 4f(x)$ and $f(1) = 4$. Find the value of f(21).

Sol.

(iii) Let 'f' be a function defined from $R^+ \to R^+$. $[f(xy)]^2 = x(f(y))^2$ for all positive numbers x and y and f(2) = 6, find the value of f(50).

Sol.

(iv) Let f(x) be a function with two properties
(a) for any two real number x and y, f(x + y)=x + f(y)
and **(b)** f(0) = 2. Find the value of f(100).

Sol.

(v) Let f(x) be function such that f(3) = 1 and f(3x) = x + f(3x – 3) for all x. Then find the value of f(300).

Sol.

11. Let 'f' be a real valued function defined for all real numbers x such that for some positive constant 'a' the equation $f(x+a)=\frac{1}{2}+\sqrt{f(x)-(f(x))^2}$ holds for all x. Prove that the function f is periodic.

Sol.

12. A function f, defined for all x, y $\in$ R is such that f(1) = 2 ; f(2) = 8 & $f(x + y) - k\,x\,y = f(x) + 2y^2$, where k is some constant. Find f(x) & show that :

$f(x + y)\, f\left(\frac{1}{x+y}\right) = k$ for $x + y \neq 0$.

Sol.

13. Prove that the function defined as, f(x) =

$$\begin{cases} e^{-\sqrt{|\ln\{x\}|}} - \{x\}^{\sqrt{\frac{1}{|\ln\{x\}|}}} & \text{where ever it exists} \\ \{x\} & \text{otherwise, then} \end{cases}$$

f(x) is odd as well as even.
(where {x} denotes the fractional part function)

Sol.

14. Let f(x) = $\dfrac{9^x}{9^x+3}$ then find the value of the sum f $\left(\dfrac{1}{2006}\right)$ + f $\left(\dfrac{2}{2006}\right)$ + f $\left(\dfrac{3}{2006}\right)$ + ... + f $\left(\dfrac{2005}{2006}\right)$.

Sol.

15. The set of real values of 'x' satisfying the equality $\left[\dfrac{3}{x}\right]+\left[\dfrac{4}{x}\right]$ = 5 (where [∗] denotes the greatest integer function) belongs to the interval $\left(a, \dfrac{b}{c}\right]$ where a, b, c ∈ N and $\dfrac{b}{c}$ is in its lowest form. Find the value of a + b + c + abc.

Sol.

16. A function f : R → R is such that $f\left(\dfrac{1-x}{1+x}\right)$ = x for all x ≠ – 1. Prove the following.

Sol.

(a) $f(f(x)) = x$

Sol.

(b) $f(1/x) = -f(x)$, x ≠ 0

Sol.

(c) $f(-x-2) = -f(x) - 2$.

Sol.

17. If f(x) = max (x, 1/x) for x > 0 where max (a, b) denotes the greater of the two real numbers a and b. Define the function g(x) = f(x).f (1/x) and plot is graph.

Sol.

18. Let f be a one-one function with domain {x, y, z} and range {1, 2, 3}. It is given that exactly one of the following statements is true and the remaining two are false. $f(x) = 1$; $f(y) \neq 1$; $f(z) \neq 2$. Determine $f^{-1}(1)$

Sol.

19. (a) A function f is defined for all positive integers and satisfies $f(1) = 2005$ and
$f(1) + f(2) + ...+ f(n) = n^2f(n)$ for all $n > 1$. Find the value of f(2004)

Sol.

(b) If a,b are positive real numbers such that $a-b= 2$, then find the smallest value of the constant L for which $\sqrt{x^2+ax}-\sqrt{x^2+bx} < L$ for all $x > 0$.

Sol.

(c) Let $f(x) = x^2 + kx$; k is real number. The set of values of k for which the equation $f(x) = 0$ and $f(f(x)) = 0$ have same real solution set.

Sol.

(d) Let $P(x) = x^6 + ax^5 + bx^4 + cx^3 + dx^2 + ex + f$ be a polynomial such that $P(1) = 1$; $P(2) = 2$; $P(3) = 3$; $P(4) = 4$; $P(5) = 5$ and $P(6) = 6$ then find the value of P(7).

Sol.

(e) Let a and b be real numbers and let
$f(x) = a \sin x + b\sqrt[3]{x} + 4, \forall\ x \in R$. If $f(\log_{10}(\log_3 10)) = 5$ then find the value of $f(\log_{10}(\log_{10} 3))$

Sol.

20. Let [x] = the greatest integer less than or equal to x. If all the values of x such that the product $\left[x-\frac{1}{2}\right]\left[x+\frac{1}{2}\right]$ is prime, belongs to the set

$[x_1, x_2) \cup [x_3, x_4)$, find the value of $x_1^2+x_2^2+x_3^2+x_4^2$.

Sol.

21. Suppose p(x) is a polynomial with integer coefficients. The remainder when p(x) is divided by x – 1 is 1 and the remainder when p(x) is divided by x – 4 is 10. If r(x) is the remainder when p(x) is divided by (x – 1) (x – 4), find the value of r (2006).

Sol.

22. Let $f : R \to R - \{3\}$ be a function with the property that there exist T > 0 such that

$f(x + T) = \frac{f(x)-5}{f(x)-3}$ for every $x \in R$. Prove that f(x) is periodic.

Sol.

23. Let $f(x) = x^{135} + x^{125} - x^{115} + x^5 + 1$. If f(x) is divided by $x^3 - x$ then the remainder is some function of x say g (x). Find the value of g(10).

Sol.

24. A is a point on the circumference of a circle. Chords AB and AC divide the area of the circle into three equal parts. If the angle BAC is the root of the equation, f(x) = 0 then find f(x).

Sol.

EXERCISE – V JEE PROBLEMS

1. If the function $f : [1, \infty) \to [1, \infty)$ is defined by $f(x) = 2^{x(x-1)}$, then $f^{-1}(x)$ is **[JEE 99, 2]**

(A) $\left(\frac{1}{2}\right)^{x(x-1)}$ (B) $\frac{1}{2}(1+\sqrt{1+4\log_2 x})$

(C) $\frac{1}{2}(1-\sqrt{1+4\log_2 x})$ (D) not defined

Sol.

2. The domain of definition of the function, y(x) given by the equation, $2^x + 2^y = 2$ is **[JEE 2000(Scr.), 1]**

(A) $0 < x \le 1$ (B) $0 \le x \le 1$ (C) $-\infty < x \le 0$ (D) $-\infty < x < 1$

Sol.

3. Given X = {1, 2, 3, 4}, find all one–one, onto mappings, $f : X \to X$ such that, $f(1) = 1$, $f(2) \ne 2$ and $f(4) \ne 4$ **[REE 2000, 3]**

Sol.

4. (a) Let $g(x) = 1 + x - [x]$ & $f(x) = \begin{cases} -1 & , \ x < 0 \\ 0 & , \ x = 0 \\ 1 & , \ x > 0 \end{cases}$.

Then for all x, f(g(x)) is equal to **[JEE 2001 (Scr.), each 1 mark]**

(A) x (B) 1 (C) f(x) (D) g(x)

where [*] denotes the greatest integer function.

Sol.

(b) If $f : [1, \infty) \to [2, \infty)$ is given by, $f(x) = x + \frac{1}{x}$,

then $f^{-1}(x)$ equals.

(A) $\frac{x+\sqrt{x^2-4}}{2}$ (B) $\frac{x}{1+x^2}$ (C) $\frac{x-\sqrt{x^2-4}}{2}$ (D) $1-\sqrt{x^2-4}$

Sol.

(c) The domain of definition of $f(x) = \frac{\log_2(x+3)}{x^2+3x+2}$ is

(A) $R \setminus \{-1, -2\}$ (B) $(-2, \infty)$

(C) $R \setminus \{-1, -2, -3\}$ (D) $(-3, \infty) \setminus \{-1, -2\}$

Sol.

(d) Let E = {1, 2, 3, 4} & F = {1, 2}. Then the number of onto functions from E to F is

(A) 14 (B) 16 (C) 12 (D) 8

Sol.

(e) Let f(x) = $\frac{\alpha x}{x+1}$, $x \neq -1$. Then for what value of α is f(f(x)) = x ?

(A) $\sqrt{2}$ (B) $-\sqrt{2}$ (C) 1 (D) – 1

Sol.

5. (a) Suppose f(x) = $(x + 1)^2$ for $x \geq -1$. If g(x) is the function whose graph is the reflection of the graph of f(x) with respect to the line y = x, then g(x) equals

[JEE. 2002 (Scr.), 3 + 3]

(A) $-\sqrt{x} - 1, x \geq 0$ (B) $\frac{1}{(x+1)^2}, x \geq -1$

(C) $\sqrt{x+1}, x \geq -1$ (D) $\sqrt{x} - 1, x \geq 0$

Sol.

(b) Let function f : R $\rightarrow$ R be defined by f(x) = 2x + sinx for $x \in$ R. Then f is

(A) one to one and onto (B) one to one but NOT onto

(C) onto but NOT one to one (D) neither one to one nor onto

Sol.

6. (a) Let f(x) = $\frac{x}{1+x}$ defined from $(0, \infty) \rightarrow [0, \infty)$ then by f(x) is **[JEE. 2003 (Scr.), 2+2]**

(A) one - one but not onto (B) one- one and onto

(C) Many one but not onto (D) Many one and onto

Sol.

(b) Range of the function f(x) = $\frac{x^2+x+2}{x^2+x+1}$ is

(A) [1, 2] (B) $[1, \infty)$ (C) $\left[2, \frac{7}{3}\right]$ (D) $\left(1, \frac{7}{3}\right]$

Sol.

7. Let f(x) = sinx + cosx, g (x) = $x^2 - 1$. Thus g(f(x)) is invertible for $x \in$ **[JEE 2004 (Scr.), 1]**

(A) $\left[-\frac{\pi}{2}, 0\right]$ (B) $\left[-\frac{\pi}{2}, \pi\right]$ (C) $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$ (D) $\left[0, \frac{\pi}{2}\right]$

Sol.

8. If the functions f(x) & g(x) are defined on $R \to R$ such that

$$f(x) = \begin{cases} 0, & x \in \text{rational} \\ x, & x \in \text{irrational} \end{cases}, \quad g(x) = \begin{cases} 0, & x \in \text{irrational} \\ x, & x \in \text{rational} \end{cases}$$

then (f – g) (x) is **[JEE 2005 (Scr.), 1]**
(A) one – one and onto (B) neither one–one nor onto
(C) one-one but not onto (D) onto but not one-one
Sol.

9. Let S = {1, 2, 3, 4}. The total number of unordered pairs of disjoint subsets of S is equal to **[JEE 2010]**
(A) 25 (B) 34 (C) 42 (D) 41
Sol.

10. Let f(x) = x^2 and g(x) = sin x for all $x \in R$. Then the set of all x satisfying (f o g o g o f) (x) = g(g o g of) (x), where (f o g) (x) = f(g(x)), is

(A) $\pm\sqrt{n\pi}$, $n \in \{0, 1, 2, ...\}$ (B) $\pm\sqrt{n\pi}$, $n \in \{1, 2, ...\}$

(C) $\frac{\pi}{2} + 2n\pi$, $n \in \{......, -2, -1, 0, 1, 2 .....\}$

(D) $2n\pi$, $n \in \{......, -2, -1, 0, 1, 2, ...\}$ **[JEE 2011]**
Sol.

11. The function f : [0, 3] $\to$ [1, 29], defined by f(x) = $2x^3 - 15x^2 + 36x + 1$, is **[JEE 2012]**
(A) one-one and onto. (B) onto but not one-one.
(C) one-one but not onto. (D) neither one-one nor onto
Sol.

12. Let f : (–1, 1) $\to$ IR be such that $f(\cos 4\theta) = \frac{2}{2 - \sec^2\theta}$

for $\theta \in \left(0, \frac{\pi}{4}\right) \cup \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$. Then the value(s) for $f\left(\frac{1}{3}\right)$ is (are)

(A) $1 - \sqrt{\frac{3}{2}}$ (B) $1 + \sqrt{\frac{3}{2}}$ (C) $1 - \sqrt{\frac{2}{3}}$ (D) $1 + \sqrt{\frac{2}{3}}$
Sol. **[JEE 2012]**

Answer Ex–I — SINGLE CORRECT (OBJECTIVE QUESTIONS)

1. D	**2.** D	**3.** B	**4.** A	**5.** A	**6.** A	**7.** D	**8.** B
9. B	**10.** D	**11.** D	**12.** A	**13.** C	**14.** B	**15.** D	**16.** C
17. C	**18.** B	**19.** A	**20.** A	**21.** B	**22.** C	**23.** B	**24.** C
25. B	**26.** D	**27.** A	**28.** A	**29.** C	**30.** C	**31.** D	**32.** D
33. D	**34.** B	**35.** C	**36.** D	**37.** C	**38.** B	**39.** B	**40.** A
41. D	**42.** D	**43.** C	**44.** B	**45.** D	**46.** D	**47.** B	**48.** D
49. B	**50.** B	**51.** C	**52.** D	**53.** C	**54.** D	**55.** A	**56.** C
57. A	**58.** A	**59.** C	**60.** C	**61.** B			

Answer Ex–II — MULTIPLE CORRECT (OBJECTIVE QUESTIONS)

1. A,B **2.** B,C,D **3.** A,C **4.** A,C **5.** A,B,C **6.** A,B,C **7.** B,C,D
8. B,C,D **9.** A,D **10.** A,B,C,D

Answer Ex–III — SUBJECTIVE QUESTIONS

1. **(i)** $R - \{-1, 1\}$ **(ii)** $(0, \infty)$ **(iii)** R **(iv)** $[-2, 0) \cup (0, 1)$ **(v)** $\left(\frac{1}{2},1\right) \cup \left(1,\frac{3}{2}\right)$

(vi) $[0, 1]$ **(vii)** $[-1, 1]$ **(viii)** R **(ix)** ϕ **(x)** $\bigcup_{n\in I}\left[n\pi, n\pi+\frac{\pi}{4}\right]$

(xi) $R - \{2n\pi\}, n \in I$ **(xii)** $(0, 1) \cup [4, 5)$ **(xiii)** $(2, 3)$

2. **(i)** $[0, \infty)$ **(ii)** $\left[-\frac{1}{2},\frac{1}{2}\right]$ **(iii)** $[0, 4]$ **(iv)** $\{-1, 1\}$ **(v)** $[0, 10]$

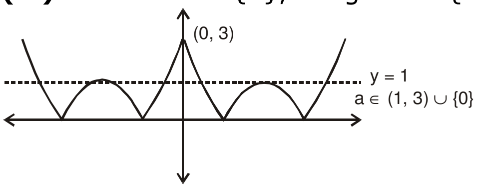
(vi) $(0, 1]$ **(vii)** $(-\infty, \frac{49}{20}]$ **(viii)** $[-4, 3]$ **(ix)** $[-1, 1]$ **(x)** $(-\infty, 1]$

(xi) R^+ **(xii)** $\left[\frac{1}{3},1\right]$ **(xiii)** $\left(-\infty,-\frac{1}{4}\right] \cup \left[-\frac{1}{20},\infty\right)$ **(xiv)** $\left[\frac{1}{3},3\right]$

(xv) $\left[0,\frac{3}{\sqrt{2}}\right]$ **(xvi)** $[4, \infty)$ **(xvii)** $[-11, 16]$ **(xviii)** $\left[\frac{3}{4},1\right]$

3. **(i)** Domain : R, Range : $\frac{1}{\sqrt{7}} \le y \le 1$ **(ii)** Domain : N ∪ {0}, Range : (n! : n = 0, 1, 2,.....}

(iii) Domain : R – {3}, Range : R – {6} **(iv)** Domain : R, Range : {1}

5. 

6. $\left\{\frac{3}{2}\right\}$ **7.** $g(x) = \begin{cases} -x & , \ -2 \le x < 0 \\ 0 & , \ 0 \le x \le 1 \\ 2(x-1) & , \ 1 < x \le 2 \end{cases}$

8. **(i)** No. **(ii)** Yes **(iii)** No **(iv)** No

9. **(i)** many–one **(ii)** many–one **(iii)** one–one **(iv)** many–one **(v)** one–one **(vi)** many–one

10. **(i)** into **(ii)** onto **(iii)** into **(iv)** onto

11. **(i)** bijective (injective as well as surjective) **(ii)** neither injective nor surjective

(iii) neither surjective nor injective **(iv)** surjective but not injective

12. **(i)** No **(ii)** Yes **(iii)** yes **(iv)** No

13. **(i)** fog = x, x > 0 ; gof = x, x ∈ R **(ii)** |sin x|, sin |x|

(iii) $\sin^{-1}(x^2)$, $(\sin^{-1} x^2)$ **(iv)** $\frac{3x^2-4x+2}{(1-x)^2}, \frac{x^2+2}{x^2+1}$

14. $f(g(x)) = \begin{cases} 2-2x+x^2 & 0 \le x \le 1 \\ 2-x & -1 \le x < 0 \end{cases}$ **15.** $(fof)(x) = \begin{cases} 2+x & , \ 0 \le x \le 1 \\ 2-x & , \ 1 < x \le 2 \\ 4-x & , \ 2 < x \le 3 \end{cases}$

16. Domain : [1, 2] ; Range : [$\ln 2$, $\ln 4$) **17.** –3 **18.** 2046

19. **(i)** $f(x) = \begin{cases} x^2 - \sin x & -1 < x \le 0 \\ -x + e^x & x \le -1 \end{cases}$ **(ii)** $f(x) = \begin{cases} -x^2 + \sin x & -1 < x \le 0 \\ x - e^x & x \le -1 \end{cases}$

20. **(i)** neither even nor odd **(ii)** even **(iii)** odd **(iv)** even **(v)** odd

21. **(i)** 2π **(ii)** 2π **(iii)** 24 **(iv)** 70π **(v)** $\frac{2\pi}{3}$ **(vi)** 2π
(vii) $\pi/2$ **(viii)** 2π

22. **(i)** π **(ii)** $\frac{2\pi}{3}$ **(iii)** 2 **(iv)** 2π **(v)** 2π **(vi)** $2^n \pi$ **(vii)** π

23. **(i)** 2p **(ii)** 8 **24.** $f^{-1} : R \to R$, $f^{-1}(x) = \frac{1}{2} \ln\left(x + \sqrt{x^2+1}\right)$

25. B = [0, 4]; $f^{-1}(x) = \frac{1}{2}\left(\sin^{-1}\left(\frac{x-2}{2}\right) - \frac{\pi}{6}\right)$ **26.** $f^{-1}(x) = x + (-1)^{x-1}$, $x \in N$

Answer Ex–IV ADVANCED SUBJECTIVE QUESTIONS

1. **(i)** $\left[-\frac{5\pi}{4},\frac{-3\pi}{4}\right] \cup \left[-\frac{\pi}{4},\frac{\pi}{4}\right] \cup \left[\frac{3\pi}{4},\frac{5\pi}{4}\right]$ **(ii)** $\left(-4,-\frac{1}{2}\right) \cup (2, \infty)$ **(iii)** $(-\infty, -3]$

(iv) $(-\infty, -1) \cup [0, \infty)$ **(v)** $(3-2\pi < x < 3-\pi)$ U $(3 < x \le 4)$

(vi) $\left(0,\frac{1}{100}\right) \cup \left(\frac{1}{100},\frac{1}{\sqrt{10}}\right)$ **(vii)** $(-1 < x < -1/2)$ U $(x > 1)$

(viii) $\left[\frac{1-\sqrt{5}}{2},0\right) \cup \left[\frac{1+\sqrt{5}}{2},\infty\right)$ **(ix)** $(-3, -1]$ U $\{0\}$ U $[1,3)$

(x) $\{4\} \cup [5, \infty)$ **(xi)** $(0, 1/4)$ U $(3/4, 1)$ U $\{x : x \in N, x \ge 2\}$

(xii) $\left(-\frac{1}{6},\frac{\pi}{3}\right] \cup \left[\frac{5\pi}{3},6\right)$ **(xiii)** $[-3, -2) \cup [3, 4)$ **(xiv)** ϕ

(xv) $2K\pi < x < (2K+1)\pi$ but $x \ne 1$ where K is non–negative integer

(xvi) $\{x \mid 1000 \le x < 10000\}$ **(xvii)** $(-2, -1) \cup (-1, 0) \cup (1, 2)$

(xviii) $(1, 2) \cup \left(2,\frac{5}{2}\right)$ **(xix)** $(-\infty, -3) \cup (-3, 1] \cup [4, \infty)$

2. **(i)** D : $x \in R$ R : $[0, 2]$ **(ii)** D = R ; range $[-1,1]$

(iii) D : $\{x \mid x \in R ;\ x \ne -3 ;\ x \ne 2\}$; R : $\{f(x) \mid f(x) \in R,\ f(x) \ne 1/5 ;\ f(x) \ne 1\}$

(iv) D : $x \in R - \{-1, 1\}$; range $\in R - \{-1, 1\}$ **(v)** D : $x \in [-1, 2]$; range $\in (-\infty, \sqrt{3}]$

(vi) D : $x \in (2n\pi, (2n+1)\pi) - \{2n\pi + \frac{\pi}{6}, 2n\pi + \frac{\pi}{2}, 2n\pi + \frac{5\pi}{6}, n \in I\}$ and

R : $\log_a 2$; $a \in (0, \infty) - \{1\} \Rightarrow$ Range is $(-\infty, \infty) - \{0\}$

(vii) D : $[-4, \infty) - \{5\}$; R : $\left(0,\frac{1}{6}\right) \cup \left(\frac{1}{6},\frac{1}{3}\right]$

3. $p \in (-4, 4) - \{0\}$

4. **(i)** $f(x) = 1$ for $x < -1$ & $-x$ for $-1 \le x \le 0$; **(ii)** $f(x) = -1$ for $x < -1$ and x for $-1 \le x \le 0$.

5. **(a)** $2K\pi \le x \le 2K\pi + \pi/2$ where $K \in I$ **(b)** $[-3/2, -1]$

6. $x = 1$ **7.** **(i)** $\frac{e^x - e^{-x}}{2}$ **(ii)** $\frac{\log_2 x}{\log_2 x - 1}$ **(iii)** $\frac{1}{2}\log\frac{1+x}{1-x}$

8. **(i)** $y = \log(10 - 10^x)$, $-\infty < x < 1$ **(ii)** $y = x/3$ when $-\infty < x < 0$ & $y = x$ when $0 \le x < +\infty$

9. **(a)** odd **(b)** even **(c)** neither odd nor even **(d)** odd **(e)** neither odd nor even

(f) even **(g)** even **(h)** even

10. **(i)** –3/4, **(ii)** 64 **(iii)** 30 **(iv)** 102 **(v)** 5050

12. $f(x) = 2x^2$ **14.** 1002.5 **15.** 20 **17.** $g(x) = \begin{cases} \frac{1}{x^2} \text{ if } 0 < x \le 1 \\ x^2 \text{ if } x > 1 \end{cases}$

18. $f^{-1}(1) = y$ **19.** **(a)** $\frac{1}{1002}$, **(b)** 1, **(c)** [0, 4), **(d)** 727, **(e)** 3

20. 11 **21.** 6016 **23.** 21 **24.** $f(x) = \sin x + x - \frac{\pi}{3}$

Answer Ex–V | JEE PROBLEMS

1. B **2.** D

3. {(1,1), (2, 3), (3, 4), (4, 2)} ; {(1,1), (2, 4), (3, 2), (4, 3)} and {(1,1), (2, 4), (3, 3), (4, 2)}

4. **(a)** B, **(b)** A, **(c)** D, **(d)** A, **(e)** D **5.** **(a)** D, **(b)** A **6.** **(a)** A, **(b)** D

7. C **8.** A **9.** D **10.** A **11.** B **12.** A,B